

Title: Wire Identification in a Multi-Wire Cable

Brief Overview:

The students will hear about a hypothetical situation in which a cable with a large number of wires runs from the basement to the 20th floor of a building. The ends of the cable are indistinguishable. The students are asked to think about how they might determine a correspondence between the wire ends in the basement and those on the 20th floor. Since the distance from the basement to the 20th floor is great (and the elevator is broken) (This is a control cable for the elevator!), they are encouraged to find a solution that minimizes the time climbing stairs. They will be guided towards a solution technique involving circuit testing with a simple battery and light arrangement. Then they will be urged to find a way to do this test minimizing travel time. The students will have an opportunity to try out their technique on a “real” cable.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

- **Number and Operations**

- Students will learn to partition an integer as a sum of integers in a way useful to solving the problem. Students may learn about “triangle numbers”, as they are the numbers for which a solution is easily obtained.

- **Process Standards**

- **Problem Solving**

- Students will develop a general technique to solve the problem, then refine it, making it more optimal.

- **Reasoning and Proof**

- Students will argue that their solution works and is optimal. They will search for and discover a pattern for partitioning the wires in the cable and testing them.

Communication

Students will describe and defend their algorithms.

Connections

This activity links with the disciplines of Logic and Science (or Physics).

Links to Maryland High School Mathematics Core Learning Goals:**Geometry, Measurement, and Reasoning**

- Students will use logic based on partial information to complete the correspondence between the wire ends at each end of the cable.

Links to National Science Education Standards:**• Physical Science**

Students will understand simple electrical circuits.

Links to Maryland High School Science Core Learning Goals:**• Concepts of Physics**

Students will understand simple electrical circuits.

Grade/Level:

Gifted 5th graders, interested middle school students and, maybe, high school students

Duration/Length:

This could probably be done in two hours, probably not on the same day.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Simple electric circuits
- Solving logic problems
- Identifying number patterns

Student Outcomes:

Students will:

- develop an algorithm to find the correspondence between the wires at two ends of a cable.
- refine and optimize that algorithm.
- test their algorithm on “real” cables.

Materials/Resources/Printed Materials:

- “Real cables” made from n segments of Bell wire (ends stripped) bound together with duck tape in such a way that the ends cannot easily be traced. All the wire for any particular cable should have the same color insulation so that the ends cannot be distinguished.
- Some sort of electrical test equipment for determining the presence of a circuit. The “resistance” function of a simple multi-meter will do. Alternatively, a simple battery and light bulb circuit will also serve.
- Tape and markers for labeling wire ends.

- Perhaps, square grids the size of the number of wires in the cable for the students to use “logic-problem fashion.”

Development/Procedures:

1. Describe the situation to the students: An elevator control cable made of 10 wires has been installed in a building and you need to connect it. It runs from the motors above the 20th floor to the control box in the basement. Unfortunately the ends of the cable are not marked; there is no color coding. You need label each end of each wire 1,2,3,...,10 so that the wire labeled 1 in the basement actually connects to the end labeled 1 above the 20th floor, etc. You have only your simple test equipment. And, you would like to do it in a way that minimizes your trips up and down 20 flights of stairs. You are working alone. What do you do?
2. Discuss this with the whole class. Describe whatever test equipment you have and show them how it works. Perhaps take a few minutes to talk about simple electrical circuits. Steer the class towards the idea of, say, twisting two wire ends together in the basement, then going upstairs and testing the wire ends there until you find the two that make a complete circuit. Discuss how they might use this information. Note: you don't yet have a match, but you've narrowed the possibilities, at least for the two wires you've been using. Discuss how you might use this information and what testing you might do next.
3. When you are satisfied they have the basic idea, divide them into groups of 3 or 4 and give them a sample cable (maybe with 3 or 6 wires) and test equipment. Tell them that they can't test both ends of the cable at the same time; they need to pretend that they're actually a long distance away from the opposite end. Ask them to attempt to label the cable, keeping track of how many trips they make “up and down the stairs.” At this point the algorithm is: twist some ends together at one end; go to the other end and make some measurements; think about what you've learned; twist some cables at that end; return to the other end; make some measurements and think about what you've learned; and repeat until you have a solution.
4. Discuss the experience with the whole class. Ask how many trips each had to make. Discuss how they might lower the number of trips. What is the lowest number of trips possible? Steer them toward the idea of only making one round trip. Discuss how to partition the wires at each end into subsets to make this possible.
5. Let the groups try their cable again to see if they can resolve the correspondence in one round trip. Help if needed. Note: an algorithm is included as attachment 1. Other algorithms work for certain numbers, e.g., triangular numbers.
6. When they have the idea for the first cable, ask how their algorithm changes if the number of wires in the cable is different. Give individuals or groups different numbers to try (on paper). I believe the “triangular numbers” {1, 3, 6, 10, 15, . . .} are easy to work with. You might take a tangent to discuss triangular numbers. Note: the algorithm is slightly different for even and odd numbers. See attachment 1.
7. Finally, give them another cable (different number of wires) and let them try their algorithm.

Assessment:

If desired, the student might be asked to label yet another “cable” as an assessment. The number of correct matches might serve as a scoring rubric.

Extension/Follow Up:

The class might try cables with even more wires to verify their algorithms and hone their logic skills. They might go on to study triangular numbers and related sequences from number theory.

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Attachment : An Algorithm

To find the correspondence between the wire ends at two ends of a multi-wire cable, if the number of wires, n , is odd:

Label the wires at one end A_1, A_2, \dots, A_n and connect them in pairs $\{A_{2i-1}, A_{2i}\}$ for $i=1, \dots, (n-1)/2$. At the other end, make sufficient electrical tests to pair the wires and label them B_1, B_2, \dots, B_n with $\{B_{2i-1}, B_{2i}\}$ corresponding to $\{A_{2j-1}, A_{2j}\}$ for some j . Of course we don't know which j yet. We do know that the unconnected B_n corresponds with (i.e., is at the opposite end of the same wire as) A_n . Now connect pairs $\{B_{2i}, B_{2i+1}\}$ for $i = 1, \dots, (n-1)/2$ (leaving B_1 unconnected). Here is a picture:

A_1	A_2	A_3	A_4	\dots	A_{n-2}	A_{n-1}	A_n
o-----o		o-----o		\dots	o-----o		o

B_1	B_2	B_3	B_4	\dots	B_{n-2}	B_{n-1}	B_n
o	o-----o		o--	\dots	----o		o-----o

Return to end A. Determine which A_j is paired with A_n (through the connections at the B end). Let $A_{j'}$ be the other member of the pair of which A_j is a member, either A_{j+1} or A_{j-1} . Checking to see which A_k is paired with $A_{j'}$ by the connections at the B end will tell us that A_k is the other end of B_{n-1} . Now do the same with $A_{k'}$. We continue in this fashion to determine all the connections.

Suppose n is even. The algorithm is similar with the following picture:

A_1	A_2	A_3	A_4	\dots	A_{n-3}	A_{n-2}	A_{n-1}	A_n
o-----o		o-----o		\dots	o-----o		o	o

B_1	B_2	B_3	B_4	\dots	B_{n-3}	B_{n-2}	B_{n-1}	B_n
o	o-----o		o--	\dots	----o		o-----o	o

Note: for some n there are other algorithms. Especially, the triangular numbers $\{1, 3, 6, 10, 15, \dots\}$ have an algorithm based on the partition, e.g., $6 = 1 + 2 + 3$. I.e., use partitions $\{A_1\}$, $\{A_2, A_3\}$, $\{A_4, A_5, A_6\}$ and $\{B_4\}$, $\{B_2, B_5\}$, $\{B_1, B_3, B_6\}$.